Compact and Malicious ***Private Set Intersection*** *for Small Sets*

Mike Rosulek, Oregon State University
Ni Trieu, Arizona State University

appeared at ACM CCS 2021

what is private set intersection (PSI)?

Alice

```
p   x   o

n   r   e

s   u   m
```

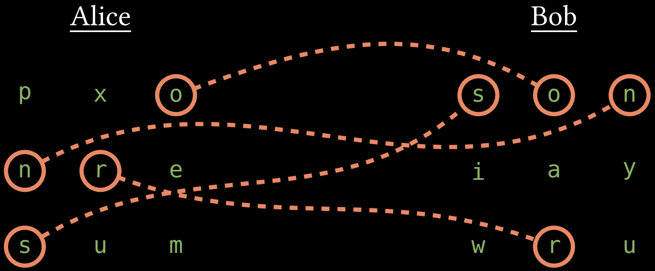
Bob

```
s   o   n

i   a   y

w   r   u
```

what is private set intersection (PSI)?



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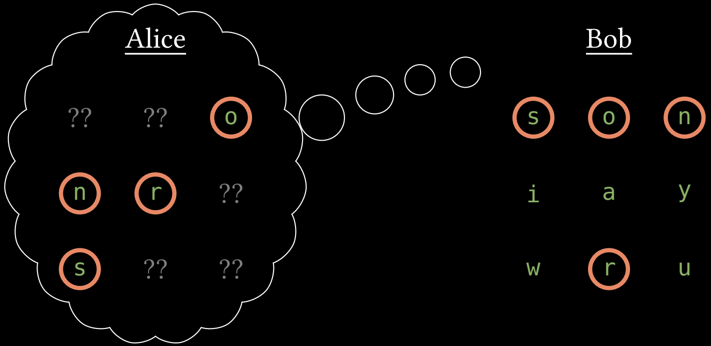


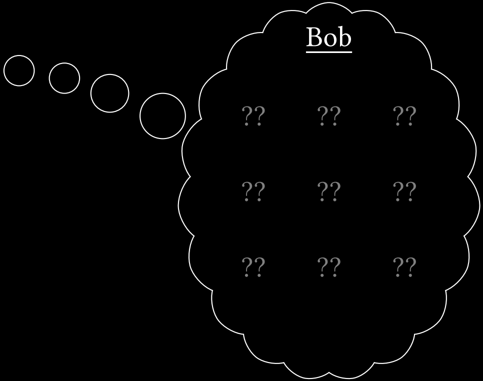
what is private set intersection (PSI)?

Alice

p x o

n r e

s u m



Bob

?? ?? ??

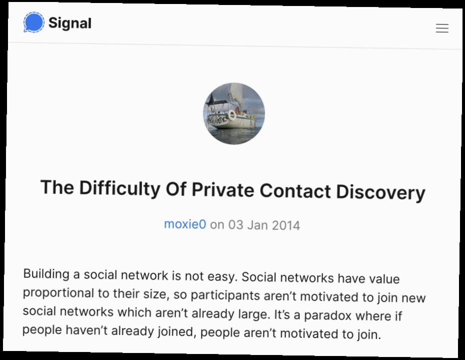
?? ?? ??

?? ?? ??

(one-sided output)

motivating PSI

motivating PSI



Signal

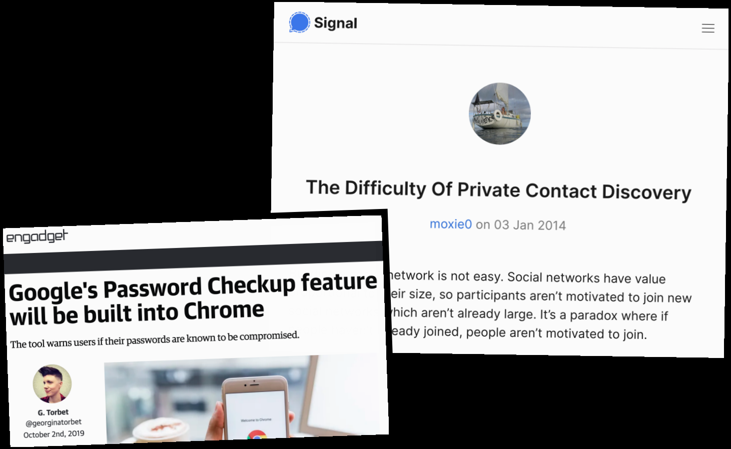
The Difficulty Of Private Contact Discovery

moxie0 on 03 Jan 2014

Building a social network is not easy. Social networks have value proportional to their size, so participants aren't motivated to join new social networks which aren't already large. It's a paradox where if people haven't already joined, people aren't motivated to join.

{my phone contacts} ∩ {users of your service}

motivating PSI



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engadget

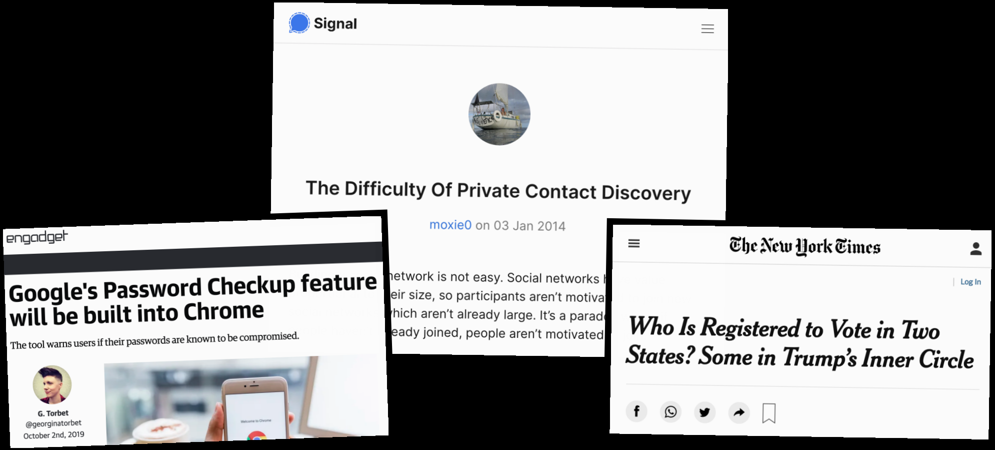
Google's Password Checkup feature will be built into Chrome

The tool warns users if their passwords are known to be compromised.

G. Torbet
@georginatorbet
October 2nd, 2019

{my passwords} ∩ {passwords found in breaches}

motivating PSI



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October 2nd, 2019

The New York Times

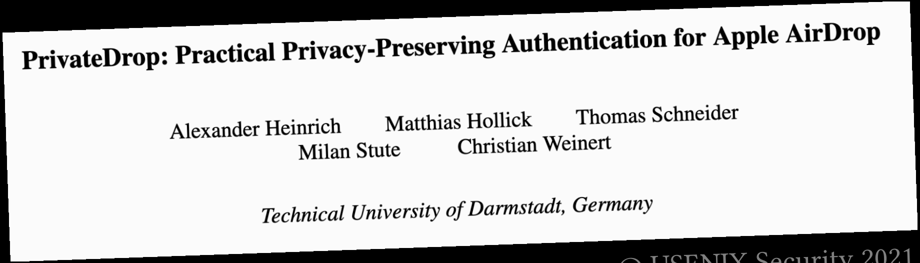
Log In

Who Is Registered to Vote in Two States? Some in Trump's Inner Circle

$$\{\text{voters registered in OR}\} \cap \{\text{voters registered in NY}\}$$

motivating PSI for small sets

motivating PSI for small sets



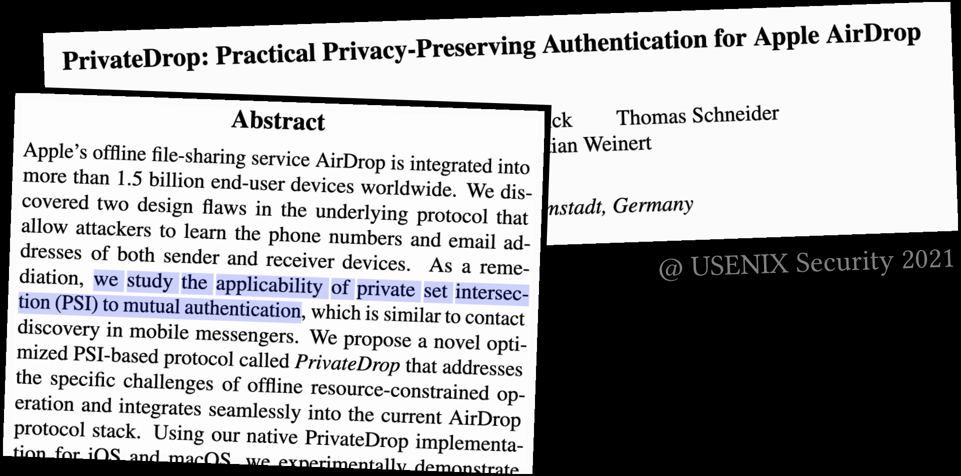
PrivateDrop: Practical Privacy-Preserving Authentication for Apple AirDrop

Alexander Heinrich Matthias Hollick Thomas Schneider
Milan Stute Christian Weinert

Technical University of Darmstadt, Germany

@ USENIX Security 2021

motivating PSI for small sets



PrivateDrop: Practical Privacy-Preserving Authentication for Apple AirDrop

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Abstract

Apple's offline file-sharing service AirDrop is integrated into more than 1.5 billion end-user devices worldwide. We discovered two design flaws in the underlying protocol that allow attackers to learn the phone numbers and email addresses of both sender and receiver devices. As a remediation, we study the applicability of private set intersection (PSI) to mutual authentication, which is similar to contact discovery in mobile messengers. We propose a novel optimized PSI-based protocol called *PrivateDrop* that addresses the specific challenges of offline resource-constrained operation and integrates seamlessly into the current AirDrop protocol stack. Using our native PrivateDrop implementation for iOS and macOS, we experimentally demonstrate

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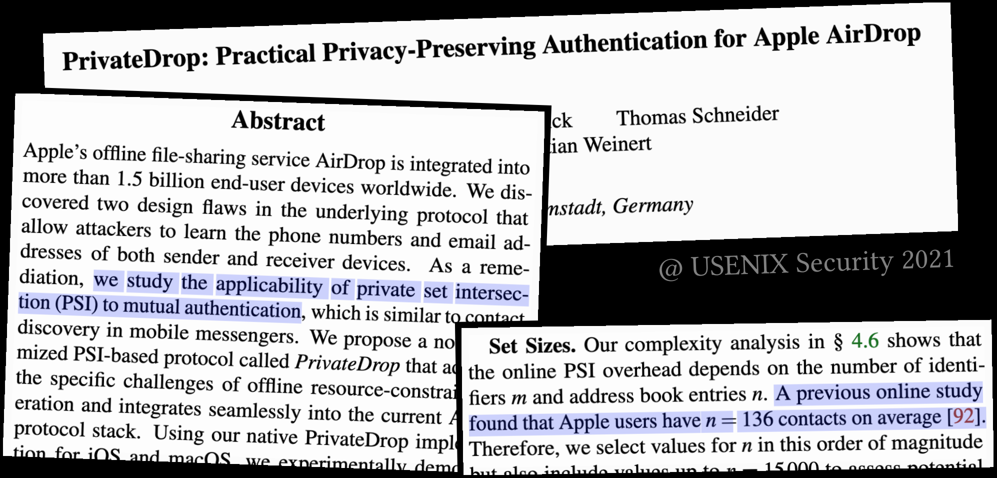
motivating PSI for small sets



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Set Sizes. Our complexity analysis in § 4.6 shows that the online PSI overhead depends on the number of identifiers m and address book entries n. A previous online study found that Apple users have $n = 136$ contacts on average [92]. Therefore, we select values for n in this order of magnitude but also include values up to $n = 15\,000$ to assess potential

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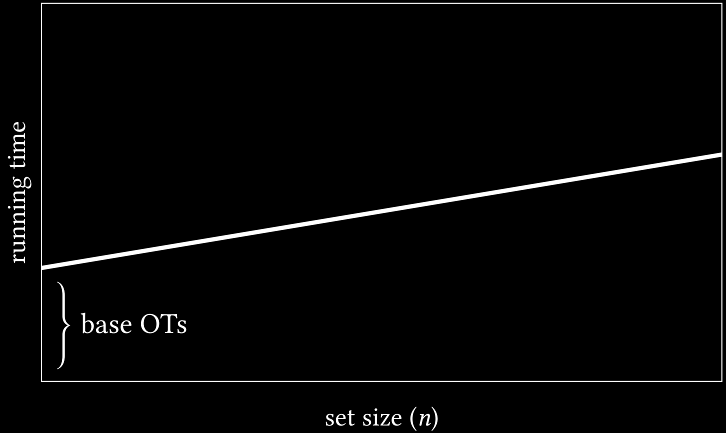
[92] = Stute, Narain, Mariotto, Heinrich, Kreitschmann, Noubir, Hollick @ *USENIX* 2019

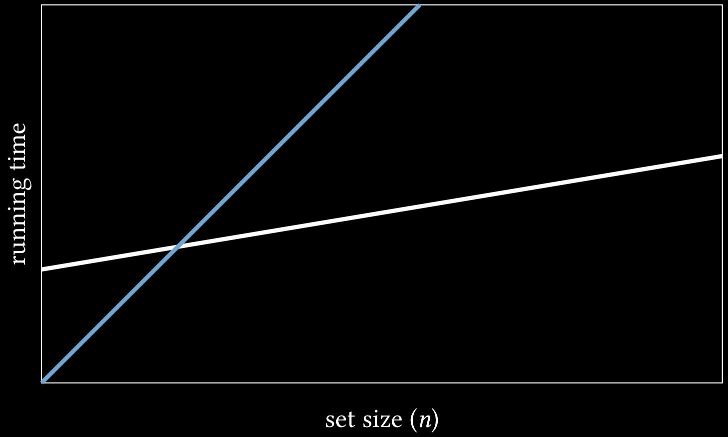
PSI techniques for small sets



OT-based PSI:

- 128 base OTs
- $O(n)$ symm-key ops

PSI techniques for small sets



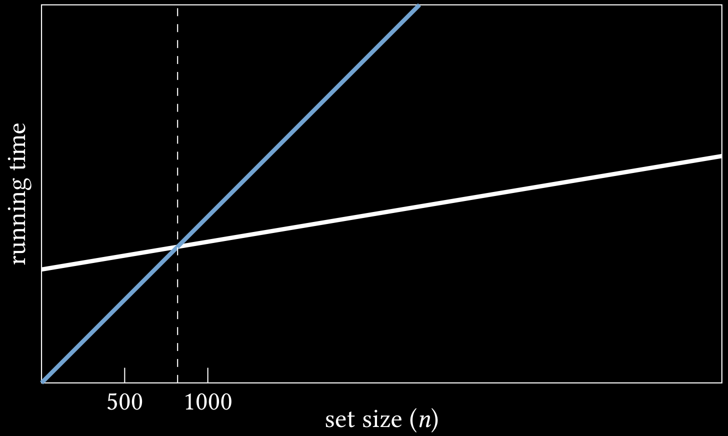
OT-based PSI:

- 128 base OTs
- $O(n)$ symm-key ops

KA-based PSI:

- $O(n)$ pub-key ops

PSI techniques for small sets



OT-based PSI:

- 128 base OTs
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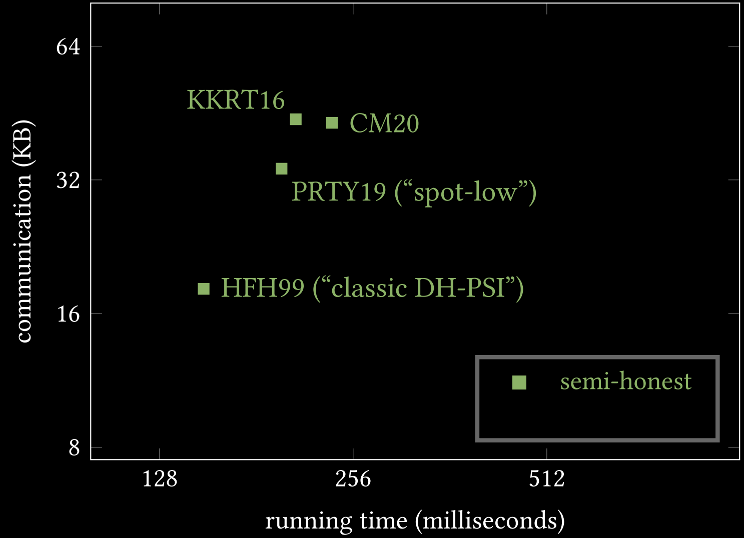
KA-based PSI:

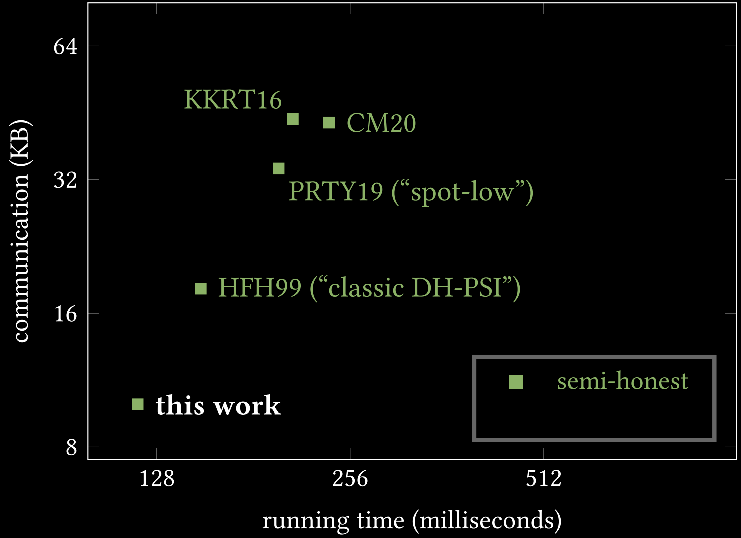
- $O(n)$ pub-key ops

PSI cost: 256 items per party:



our **semi-honest** PSI:

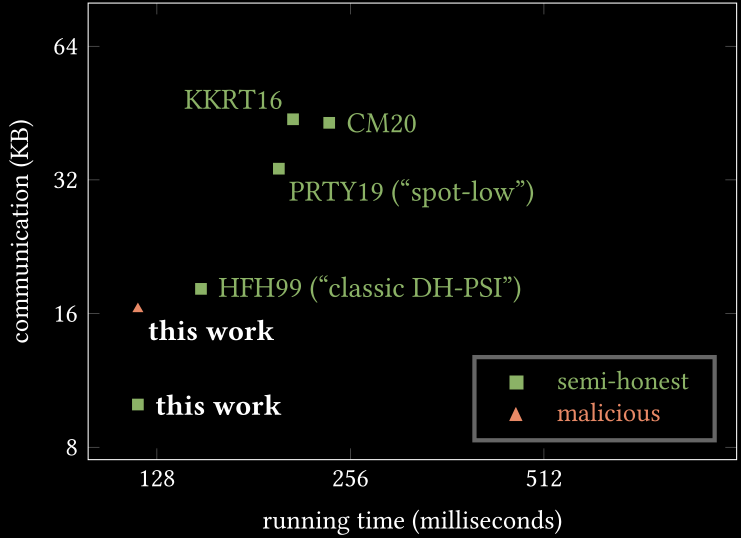
PSI cost: 256 items per party:



our **semi-honest** PSI:

- 45% ↓ communication
- 20% ↓ runtime

PSI cost: 256 items per party:



our **semi-honest** PSI:

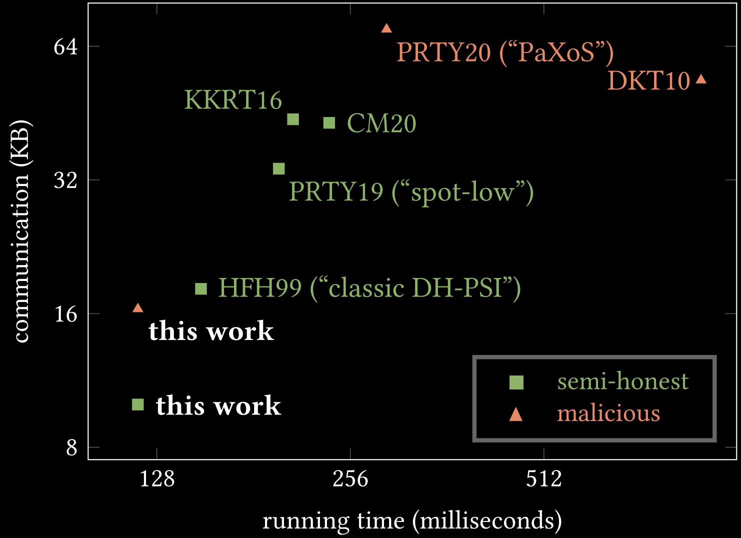
- 45% ↓ communication
- 20% ↓ runtime

our **malicious** PSI:

- 10% ↓ communication
- 20% ↓ runtime

vs. best *semi-honest* PSI!

PSI cost: 256 items per party:



our **semi-honest** PSI:

- 45% ↓ communication
- 20% ↓ runtime

our **malicious** PSI:

- 10% ↓ communication
- 20% ↓ runtime

vs. best *semi-honest* PSI!

- 75% ↓ communication
- 55% ↓ runtime

vs. best malicious PSI

✓ 1 *what is PSI?*

✓ 2 *summary of our results*

3 *review "classic DH-PSI"*

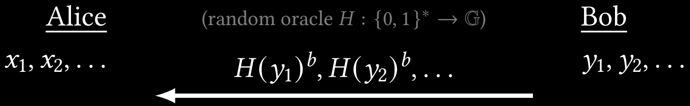
4 *our new protocol ideas*

5 *some fine print*

Alice (random oracle $H : \{0,1\}^* \to \mathbb{G}$) Bob

$x_1, x_2, \ldots$ $y_1, y_2, \ldots$

[HubermanFranklinHogg99]



Alice

(random oracle $H : \{0,1\}^* \to \mathbb{G}$)

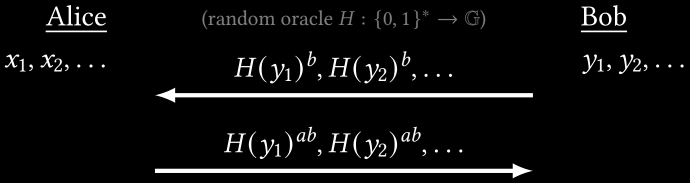
Bob

$x_1, x_2, \ldots$

$H(y_1)^b, H(y_2)^b, \ldots$

$y_1, y_2, \ldots$

[HubermanFranklinHogg99]



Alice (random oracle $H : \{0,1\}^* \to \mathbb{G}$) Bob

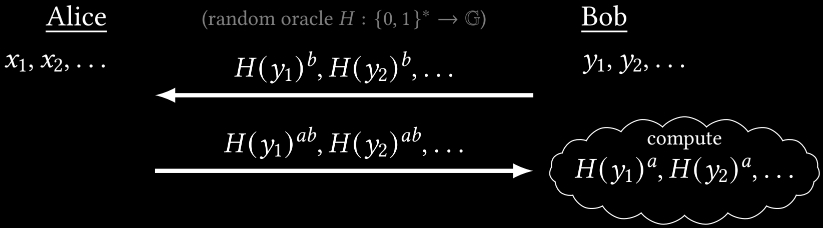
$x_1, x_2, \dots$

$y_1, y_2, \dots$

$$\xleftarrow{H(y_1)^b, H(y_2)^b, \dots}$$

$$\xrightarrow{H(y_1)^{ab}, H(y_2)^{ab}, \dots}$$

[HubermanFranklinHogg99]



Alice

(random oracle $H : \{0,1\}^* \rightarrow \mathbb{G}$)

Bob

$x_1, x_2, \ldots$

$y_1, y_2, \ldots$

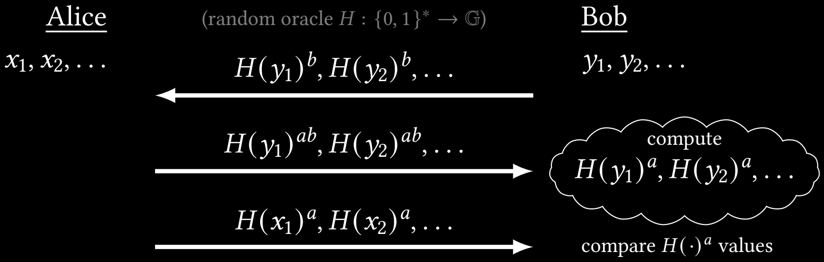
$H(y_1)^b, H(y_2)^b, \ldots$

compute

$H(y_1)^a, H(y_2)^a, \ldots$

$H(y_1)^{ab}, H(y_2)^{ab}, \ldots$

[HubermanFranklinHogg99]



Alice

(random oracle $H : \{0,1\}^* \to \mathbb{G}$)

Bob

$x_1, x_2, \ldots$

$y_1, y_2, \ldots$

$H(y_1)^b, H(y_2)^b, \ldots$

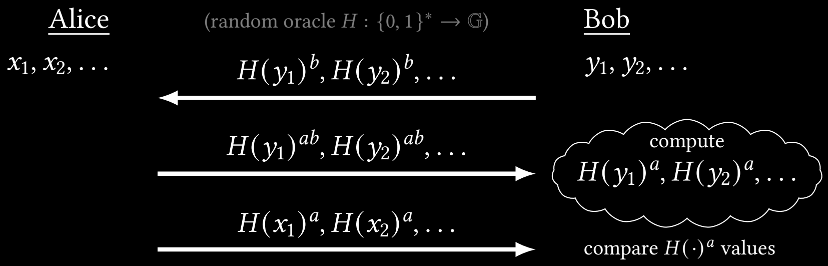
$H(y_1)^{ab}, H(y_2)^{ab}, \ldots$

compute $H(y_1)^a, H(y_2)^a, \ldots$

$H(x_1)^a, H(x_2)^a, \ldots$

compare $H(\cdot)^a$ values

[HubermanFranklinHogg99]



Alice

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Bob

$x_1, x_2, \dots$

$y_1, y_2, \dots$

$H(y_1)^b, H(y_2)^b, \dots$

$H(y_1)^{ab}, H(y_2)^{ab}, \dots$

compute $H(y_1)^a, H(y_2)^a, \dots$

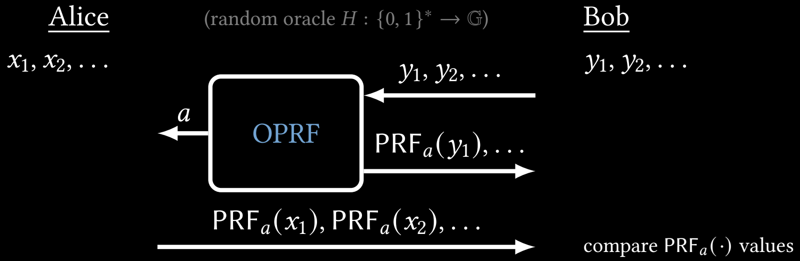
$H(x_1)^a, H(x_2)^a, \dots$

compare $H(\cdot)^a$ values

Semi-honest security:

- $x \mapsto H(x)^a$ is a PRF (DDH assumption + random oracle)
- first two messages are an oblivious PRF protocol

[HubermanFranklinHogg99]



Alice

(random oracle $H : \{0,1\}^* \to \mathbb{G}$)

Bob

$x_1, x_2, \ldots$

$y_1, y_2, \ldots$

$y_1, y_2, \ldots$

a

OPRF

$\mathsf{PRF}_a(y_1), \ldots$

$\mathsf{PRF}_a(x_1), \mathsf{PRF}_a(x_2), \ldots$

compare $\mathsf{PRF}_a(\cdot)$ values

Semi-honest security:

- $x \mapsto H(x)^a$ is a PRF (DDH assumption + random oracle)
- first two messages are an oblivious PRF protocol
- standard OPRF→PSI paradigm [FreedmanIshaiPinkasReingold05]

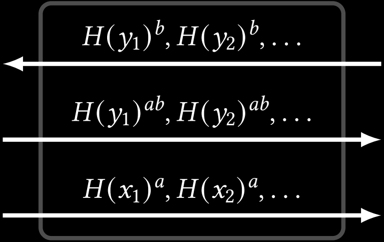
[HubermanFranklinHogg99]

Alice

$x_1, x_2, \ldots$

Bob

$y_1, y_2, \ldots$

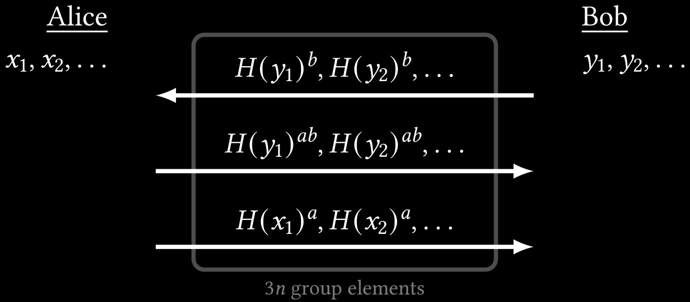


$H(y_1)^b, H(y_2)^b, \ldots$

$H(y_1)^{ab}, H(y_2)^{ab}, \ldots$

$H(x_1)^a, H(x_2)^a, \ldots$

$3n$ group elements



Alice

$x_1, x_2, \ldots$

Bob

$y_1, y_2, \ldots$

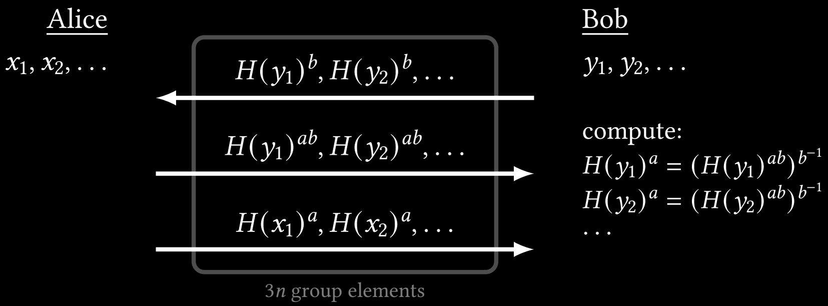
$H(y_1)^b, H(y_2)^b, \ldots$

$H(y_1)^{ab}, H(y_2)^{ab}, \ldots$

$H(x_1)^a, H(x_2)^a, \ldots$

$3n$ group elements

how could you possibly reduce communication?



Alice

$x_1, x_2, \ldots$

Bob

$y_1, y_2, \ldots$

$H(y_1)^b, H(y_2)^b, \ldots$

$H(y_1)^{ab}, H(y_2)^{ab}, \ldots$

$H(x_1)^a, H(x_2)^a, \ldots$

$3n$ group elements

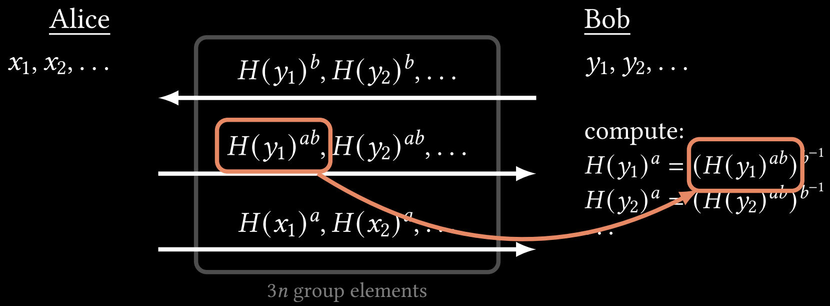
compute:

$H(y_1)^a = (H(y_1)^{ab})^{b^{-1}}$

$H(y_2)^a = (H(y_2)^{ab})^{b^{-1}}$

$\ldots$

how could you possibly reduce communication?



Alice

$x_1, x_2, \ldots$

Bob

$y_1, y_2, \ldots$

$H(y_1)^b, H(y_2)^b, \ldots$

$\boxed{H(y_1)^{ab}}, H(y_2)^{ab}, \ldots$

$H(x_1)^a, H(x_2)^a, \ldots$

compute:

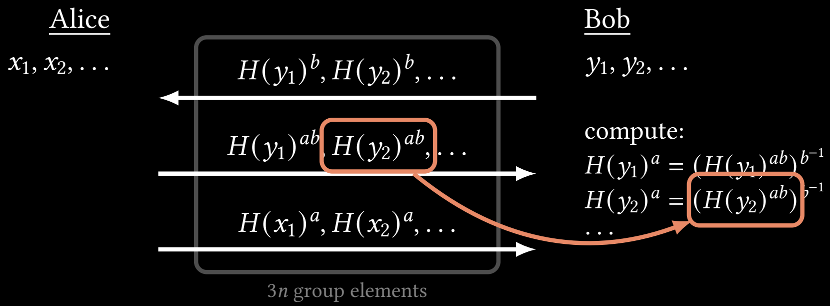
$H(y_1)^a = \boxed{(H(y_1)^{ab})}^{b^{-1}}$

$H(y_2)^a = (H(y_2)^{ab})^{b^{-1}}$

$\ldots$

$3n$ group elements

how could you possibly reduce communication?



Alice

$x_1, x_2, \ldots$

Bob

$y_1, y_2, \ldots$

$H(y_1)^b, H(y_2)^b, \ldots$

$H(y_1)^{ab}, \boxed{H(y_2)^{ab}}, \ldots$

$H(x_1)^a, H(x_2)^a, \ldots$

compute:

$H(y_1)^a = (H(y_1)^{ab})^{b^{-1}}$

$H(y_2)^a = \boxed{(H(y_2)^{ab})}^{b^{-1}}$

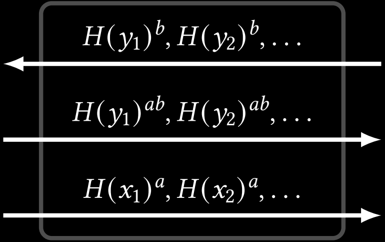
$\ldots$

$3n$ group elements

how could you possibly reduce communication?

Alice

$x_1, x_2, \ldots$



$H(y_1)^b, H(y_2)^b, \ldots$

$H(y_1)^{ab}, H(y_2)^{ab}, \ldots$

$H(x_1)^a, H(x_2)^a, \ldots$

3n group elements

Bob

$y_1, y_2, \ldots$

$H(y_1)^r = H(y_2)$

compute:

$H(y_1)^a = (H(y_1)^{ab})^{b^{-1}}$

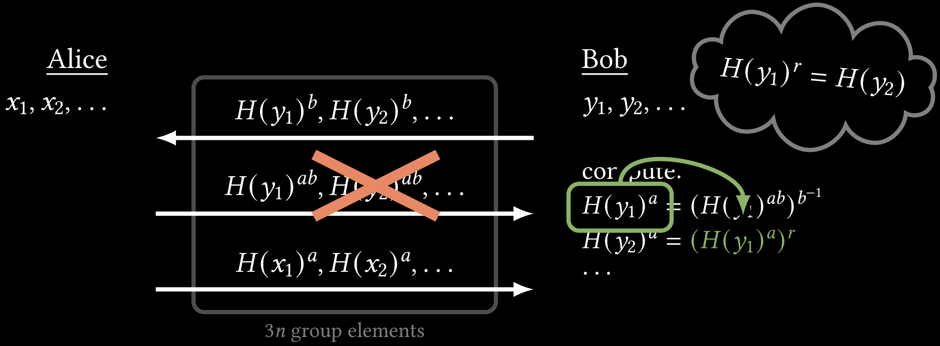
$H(y_2)^a = (H(y_2)^{ab})^{b^{-1}}$

$\ldots$

how could you possibly reduce communication?

replace random oracle with some "trapdoored" function

... where Bob knows dlog relationships between outputs



Alice

$x_1, x_2, \ldots$

Bob

$y_1, y_2, \ldots$

$H(y_1)^r = H(y_2)$

$H(y_1)^b, H(y_2)^b, \ldots$

$\cancel{H(y_1)^{ab}, H(y_2)^{ab}, \ldots}$

$H(x_1)^a, H(x_2)^a, \ldots$

3n group elements

compute:

$\boxed{H(y_1)^a} = (H(y_1)^{ab})^{b^{-1}}$

$H(y_2)^a = (H(y_1)^a)^r$

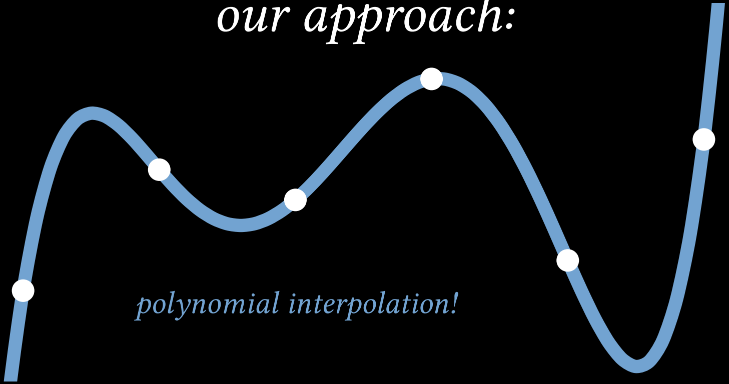
$\ldots$

how could you possibly reduce communication?

replace random oracle with some "trapdoored" function

... where Bob knows dlog relationships between outputs

our approach:



polynomial interpolation!

Alice

$x_1, x_2, \ldots$

Bob

$y_1, y_2, \ldots$

Alice

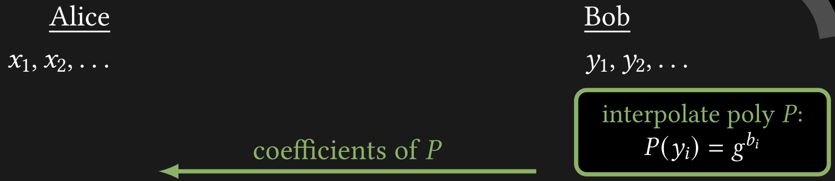
$x_1, x_2, \ldots$

Bob

$y_1, y_2, \ldots$

interpolate poly P:

$P(y_i) = g^{b_i}$



Alice

$x_1, x_2, \ldots$

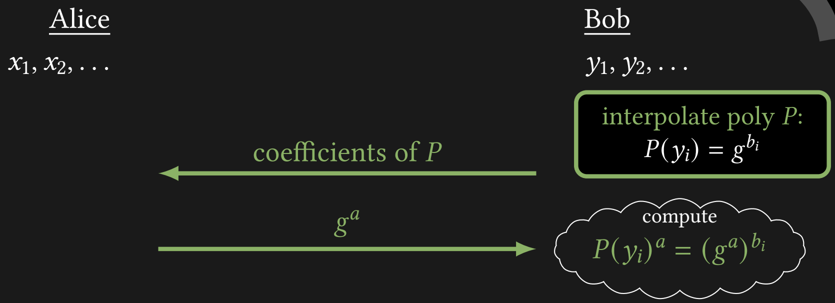
Bob

$y_1, y_2, \ldots$

interpolate poly P:

$P(y_i) = g^{b_i}$

coefficients of P



Alice

$x_1, x_2, \ldots$

Bob

$y_1, y_2, \ldots$

interpolate poly P:

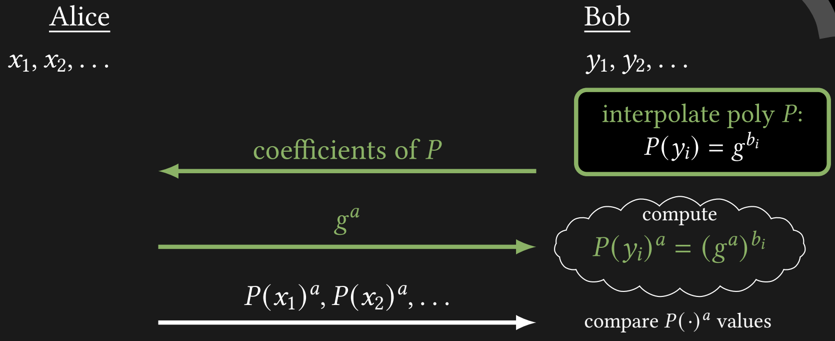
$P(y_i) = g^{b_i}$

coefficients of P

g^a

compute

$P(y_i)^a = (g^a)^{b_i}$



Alice

$x_1, x_2, \ldots$

Bob

$y_1, y_2, \ldots$

interpolate poly P:

$P(y_i) = g^{b_i}$

coefficients of P

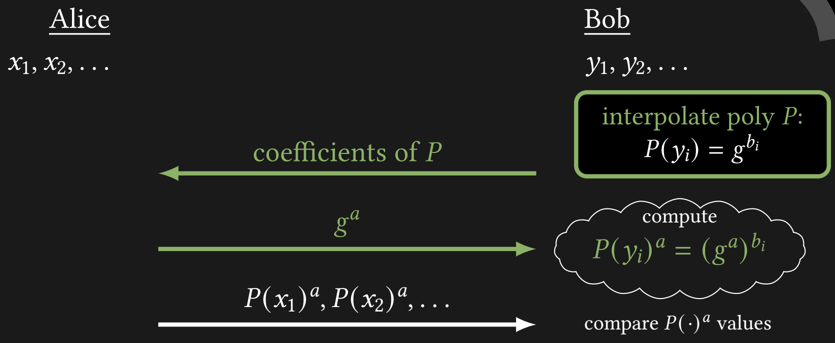
g^a

compute

$P(y_i)^a = (g^a)^{b_i}$

$P(x_1)^a, P(x_2)^a, \ldots$

compare $P(\cdot)^a$ values



Alice

$x_1, x_2, \ldots$

Bob

$y_1, y_2, \ldots$

interpolate poly P:
$P(y_i) = g^{b_i}$

coefficients of P

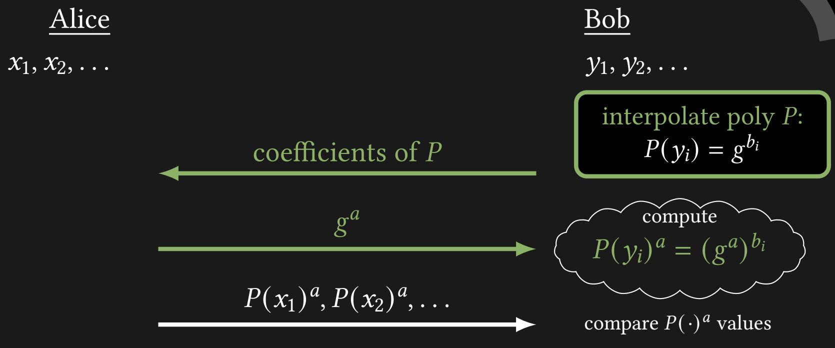
g^a

compute
$P(y_i)^a = (g^a)^{b_i}$

$P(x_1)^a, P(x_2)^a, \ldots$

compare $P(\cdot)^a$ values

correctness: Bob knows dlog of $P(y)$ for programmed points ✓



Alice

$x_1, x_2, \ldots$

Bob

$y_1, y_2, \ldots$

interpolate poly P:
$P(y_i) = g^{b_i}$

coefficients of P ←

g^a →

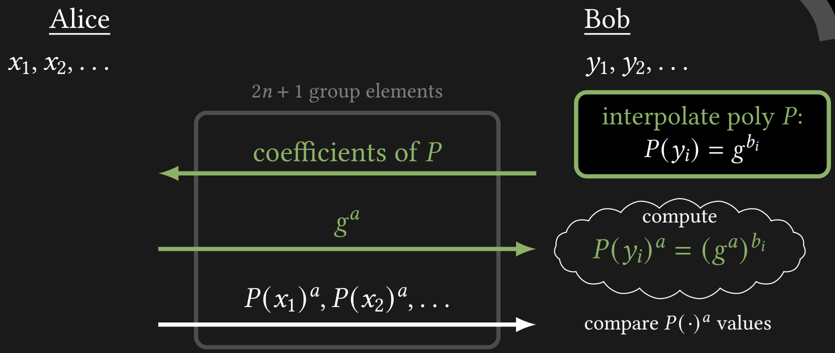
compute
$P(y_i)^a = (g^a)^{b_i}$

$P(x_1)^a, P(x_2)^a, \ldots$ →

compare $P(\cdot)^a$ values

correctness: Bob knows dlog of $P(y)$ for programmed points ✓

obliviousness: description of P doesn't leak choice of programmed points ✓



Alice

$x_1, x_2, \ldots$

Bob

$y_1, y_2, \ldots$

$2n+1$ group elements

interpolate poly P: $P(y_i) = g^{b_i}$

coefficients of P

g^a

compute $P(y_i)^a = (g^a)^{b_i}$

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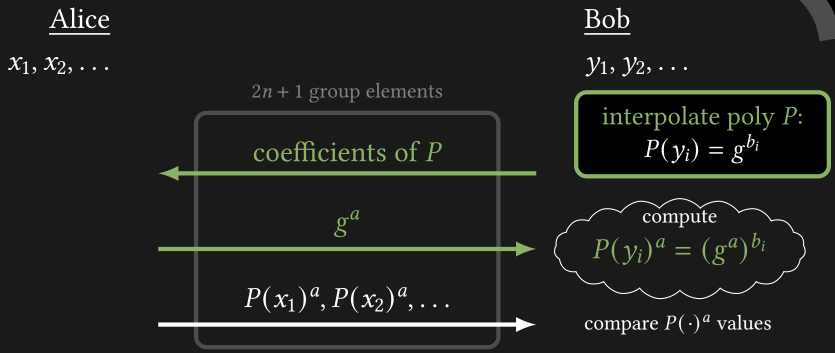

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compare $P(\cdot)^a$ values

correctness:	Bob knows dlog of $P(y)$ for programmed points ✓
obliviousness:	description of P doesn't leak choice of programmed points ✓
efficiency:	$\vert$description of $P\vert = n$ group elements ✓
$P(\cdot)^a$ is PRF:	Bob **cannot know** dlog of any *other* $P(x)$??

interpolate so that:

$P(y_i) = g^{b_i}$

$? \; ? \; \Downarrow \; ? \; ?$

other $P(x)$ outputs have unknown dlog

interpolate so that:

$$P(y_i) = g^{b_i}$$

? ? $\Downarrow$? ?

other $P(x)$ outputs
have unknown dlog

Ideal permutation model: all parties have oracle access to random Π, Π^{-1}

interpolate so that:

$P(y_i) = g^{b_i}$

? ? $\Downarrow$? ?

other $P(x)$ outputs
have unknown dlog

interpolate so that:

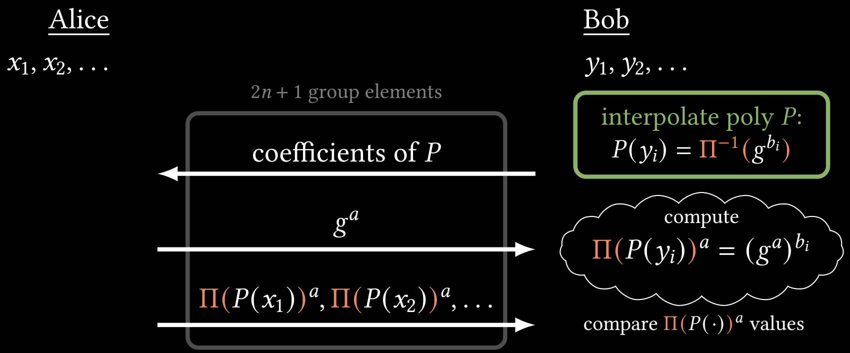
$P(y_i) = \Pi^{-1}(g^{b_i})$

$\Downarrow$ ✓

simulator can **program**
other $\Pi(P(x))$ outputs

Ideal permutation model: all parties have oracle access to random Π, Π^{-1}

our real protocol:



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compute
$\Pi(P(y_i))^a = (g^a)^{b_i}$

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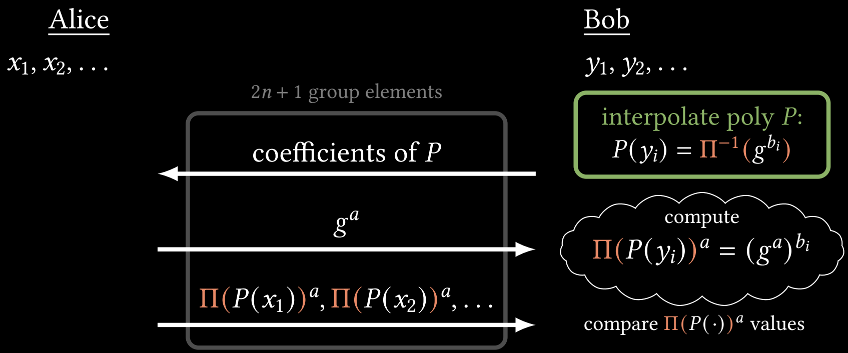
compare $\Pi(P(\cdot))^a$ values

our real protocol (fine print):



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$y_1, y_2, \ldots$

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$P(y_i) = \Pi^{-1}(g^{b_i})$

$2n + 1$ group elements

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semi-honest: Alice's group elements can be truncated

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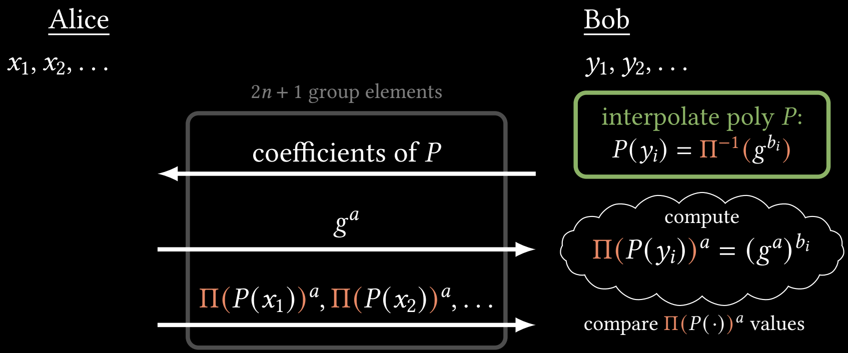

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$\Pi(P(x_1))^a, \Pi(P(x_2))^a, \ldots$

compare $\Pi(P(\cdot))^a$ values

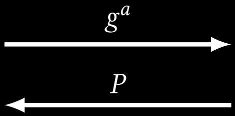
semi-honest: Alice’s group elements can be truncated

malicious: a few more strategic RO calls (to help simulator extract) *more fine print...*

$$\xrightarrow{g^a}$$

$$\xleftarrow{P}$$

$$P(y_i) = g^{b_i}$$



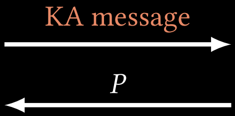
?

$$P(y_i) = g^{b_i} \quad ?$$

finite field

?

cyclic group

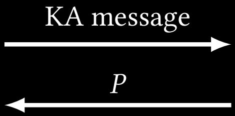


KA message

P

$$P(y_i) = \text{KA response}$$

- use generic key agreement in place of g^a, g^b



$$P(y_i) = \text{KA response}$$

- use generic key agreement in place of g^a, g^b
- KA protocol messages must be *pseudorandom bit strings*
- e.g., elliptic curve Diffie-Hellman with elligator encoding scheme [BernsteinHamburgKrasnovaLange13]

[ChoDachmanSoledJarecki16] PSI:

our protocol:

[ChoDachmanSoledJarecki16] PSI:

interpolate polynomial P so that:

$$P(y) = \boxed{\text{next message in private equality test protocol}}$$

our protocol:

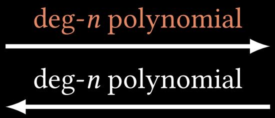
interpolate polynomial P so that:

$$P(y) = \boxed{\text{next message in key agreement protocol}}$$

[ChoDachmanSoledJarecki16] PSI:

interpolate polynomial P so that:

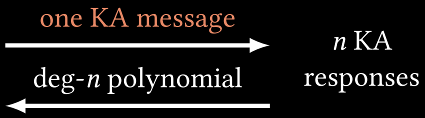
$$P(y) = \boxed{\begin{array}{l}\text{next message in private}\\ \text{equality test protocol}\end{array}}$$



our protocol:

interpolate polynomial P so that:

$$P(y) = \boxed{\begin{array}{l}\text{next message in key}\\ \text{agreement protocol}\end{array}}$$



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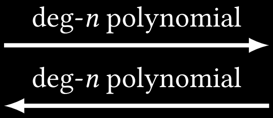

ideal cipher model

our protocol:

interpolate polynomial P so that:

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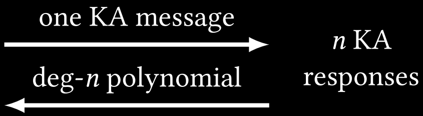
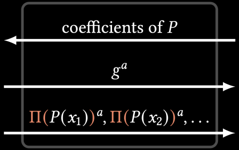


ideal permutation model

take-home message

new PSI protocol

$2n + 1$ group elements



coefficients of P

g^a

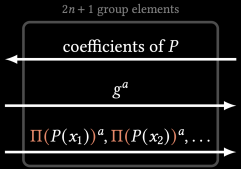
$\Pi(P(x_1))^a, \Pi(P(x_2))^a, \ldots$

ideal permutation + random oracle

full version @ `ia.cr/2021/1159`

take-home message

new PSI protocol



$2n+1$ group elements

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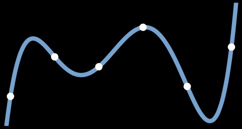
main protocol idea



interpolate poly P so that

$P(y)$ = key agreement message

full version @ `ia.cr/2021/1159`

take-home message

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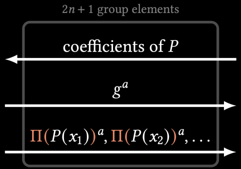

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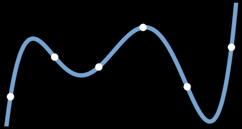

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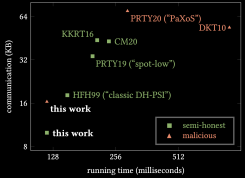
performance on small (< 1000) sets



best by good margin

full version @ ia.cr/2021/1159

take-home message

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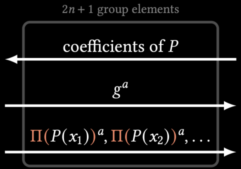

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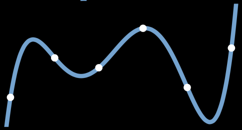

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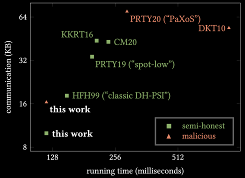

best by good margin

vs. 20-year old classic DH-PSI

faster ✓

less communication ✓

stronger security ✓

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