Homomorphic Encryption with CCA Security

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Opposing Demands for Encryption

Computational Features

Ciphertexts are active objects:

- Message homomorphism
- Proxy re-encryption
- Keyword search
- Attribute-/identity-based

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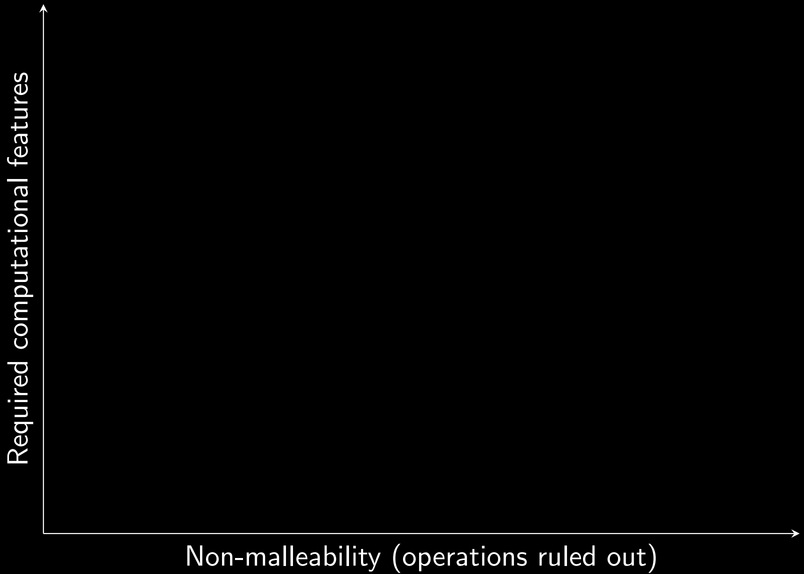
Non-malleability

Require lack "unexpected operations" an adversary may exploit

A Map of Encryption Requirements



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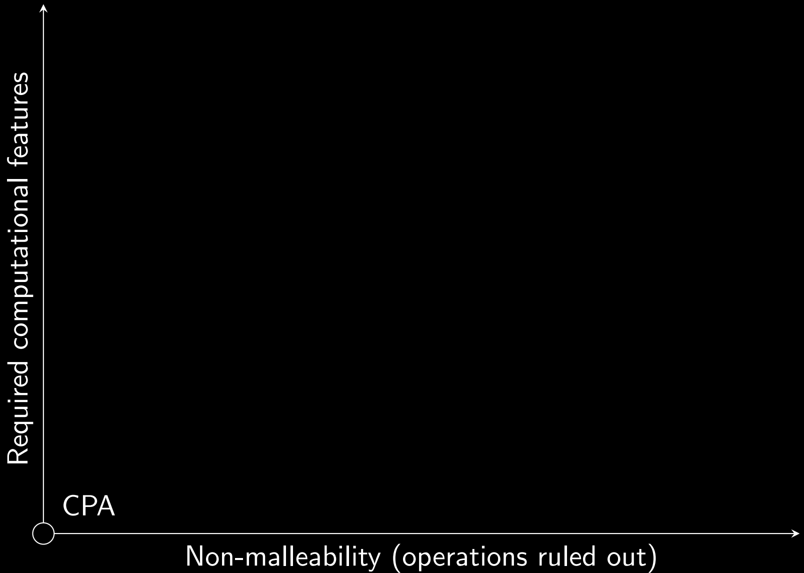

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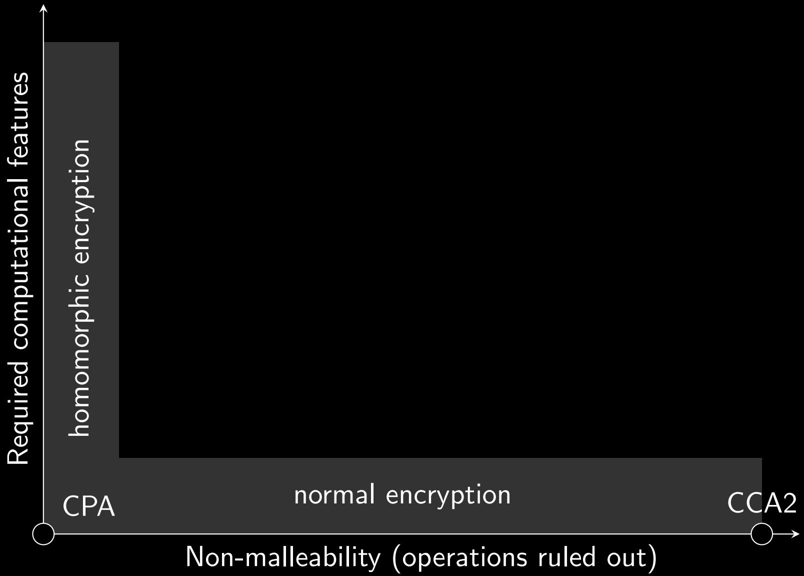

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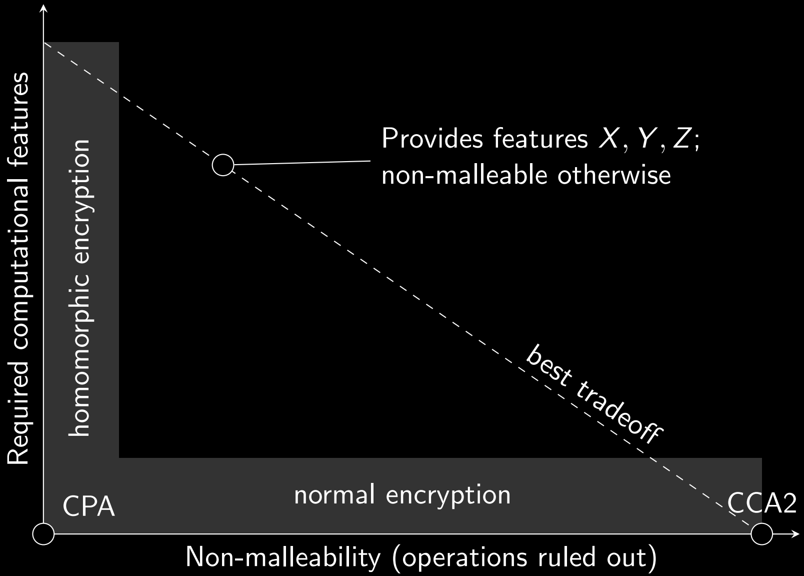

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Non-malleability is traditionally all (CCA) or nothing (CPA)

Desired Security Requirement

Scheme is *non-malleable, except for explicitly allowed features.*

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In this work:

- Address problem in context of homomorphic encryption
- New general-purpose non-malleability definition
- New family of constructions

Unary Homomorphic Encryption

Desired features:

- Anyone can change $\mathsf{Enc}(m)$ into fresh $\mathsf{Enc}(f(m))$.
- Scheme parameterized by set of allowed f's

Example: Rerandomizable Replayable-CCA (RCCA) [CKN03,G04,PR07]:

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Example: Only allowed f's are group operations $\alpha \rightsquigarrow \beta\alpha$:

- Possible to change any message to any other message
- Infeasible to change $\mathsf{Enc}(\alpha)$ into $\mathsf{Enc}(\alpha^k)$
- Infeasible to change $\mathsf{Enc}(\alpha), \mathsf{Enc}(\beta)$ into $\mathsf{Enc}(\alpha\beta)$

Outline

Security Definitions – Approach

We define security with two complementary definitions:

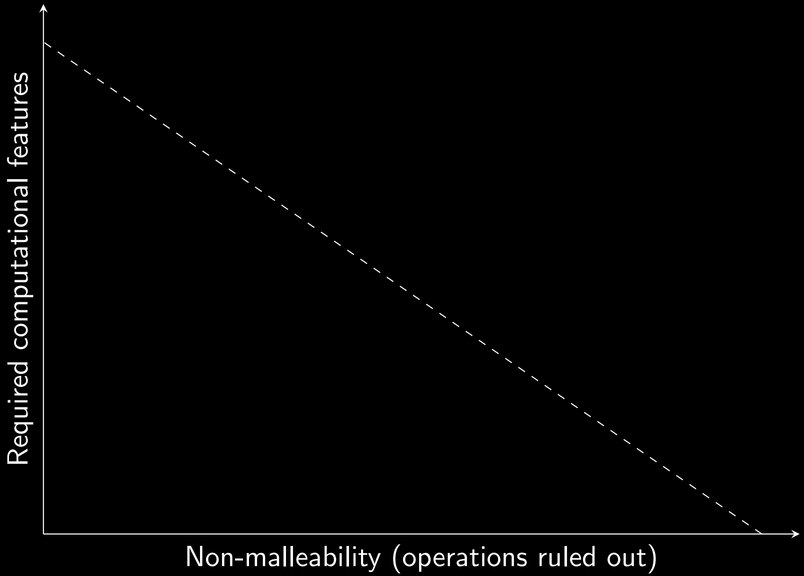
Homomorphic-CCA (HCCA) security

Scheme is non-malleable, except possibly via unary operations $f \in \mathcal{F}$

Unlinkability

One can transform $\mathsf{Enc}(m)$ to "fresh" $\mathsf{Enc}(f(m))$ for any $f \in \mathcal{F}$, as a feature of the scheme.

Security Definitions – Approach



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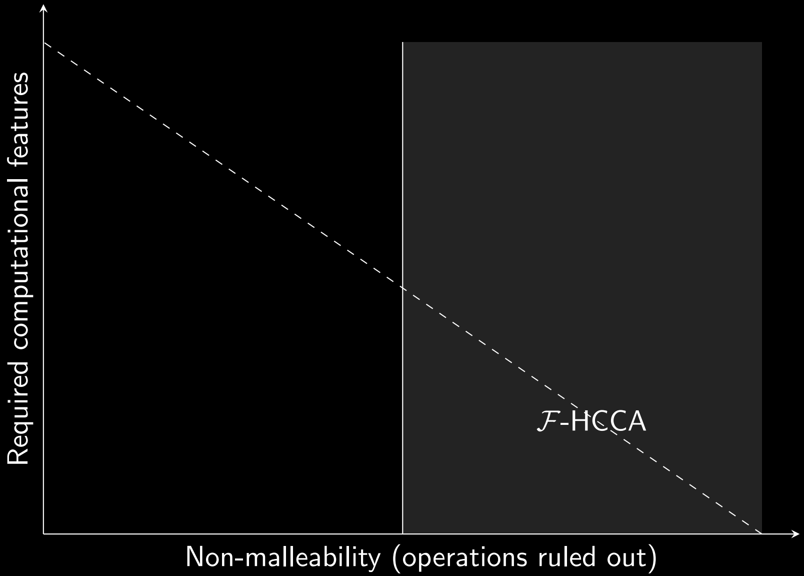

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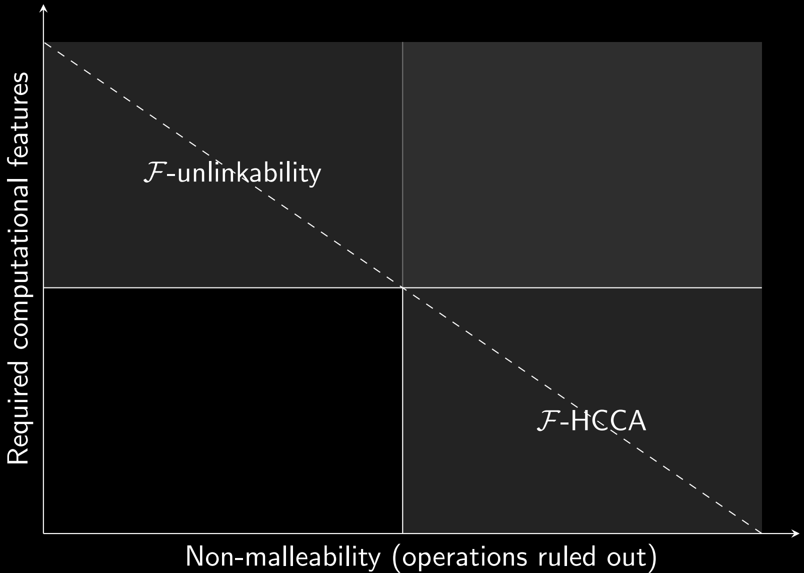

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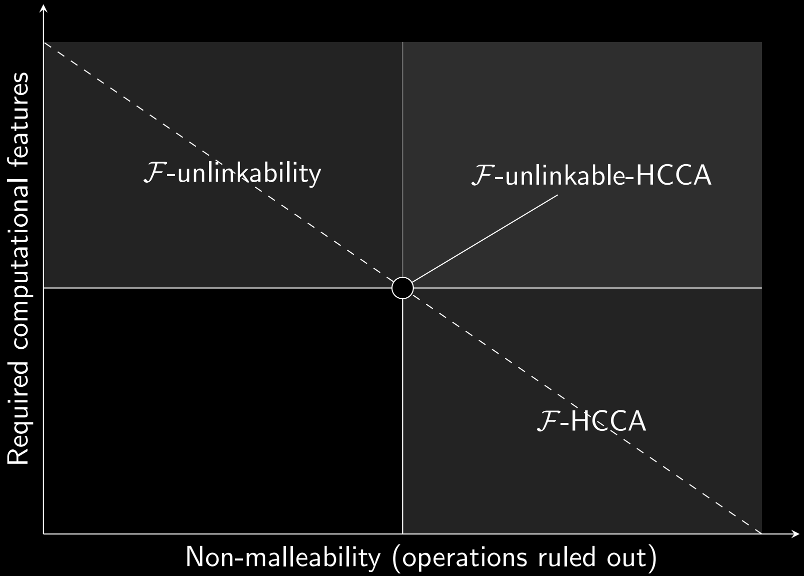

Generalizing CCA to HCCA

Start by modifying CCA experiment:

1. Generate keypair, give PK.
2. Provide Dec oracle.
3. Adversary chooses m_0, m_1.
4. Give $C \leftarrow \text{Enc}(m_0)$.
5. Provide Dec oracle, except:
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Idea for Generalization

Dec oracle should compensate for derivatives of C.

Derivative Ciphertexts

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Ciphertexts that could have been *legitimately* derived from C (i.e., via scheme's allowed features).

Different security levels for different derivative condition:

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gCCA: C' is derivative iff $R(C', C) = 1$ [S01, ADR02]

RCCA: C' is derivative iff $\mathrm{Dec}(C') = \mathrm{Dec}(C)$ [CKN03]

Can We Always Identify Derivative Ciphertexts?

For certain $\mathcal{F}$, these distributions could be identical:

- Enc(β) obtained by encrypting known β
- Enc(β) derived by legitimately multiplying Enc(α) by β/α

Problem:

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What we want:

- Ciphertexts derived from C have different distribution than independently encrypted ciphertexts

Rigged Ciphertexts

Key idea: C need not be actual encryption of some m_1:

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"Rigged" Ciphertexts

Challenge "ciphertext" can have embedded tracking information.
Extraction procedure determines how C' derived from C.

Interpreting Security Guarantee

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$\implies$ this malleability "looks like" $m \rightsquigarrow f(m)$

A Limit on Malleability

Suppose RigExtract never outputs f':

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- In particular, operations that nontrivially *combine multiple* ciphertexts.

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⋆ Homomorphic-CCA (HCCA) Security

Scheme is non-malleable except for unary operations $f \in \mathcal{F}$ if there is a good $(\mathsf{RigEnc}, \mathsf{RigExtract})$, where $\mathsf{range}(\mathsf{RigExtract}) \subseteq \mathcal{F}$.

Disclaimer:

- Oracles for RigEnc and RigExtract should be provided, too.

Outline

Relationships with other definitions

Theorem

CCA, gCCA, RCCA are all special cases of HCCA

In each of these cases:

- The only allowed transformation is identity function
- RigEnc simply uses Enc honestly

HCCA more expressive when its full power is used.

Natural UC Security Definition

Theorem

HCCA and unlinkability imply UC-secure protocol for "natural" ideal functionality

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In our UC functionality:, parties post messages, represented as "formal ciphertexts"

- **Message privacy:** Formal ciphertexts reveal nothing; only recipient can obtain underlying message
- **Homomorphic feature:** Anyone can generate a "derived post" by giving f and existing ciphertext
- **Unlinkability:** Same internal behavior for both kinds of posts
- **Non-malleability:** No one can use unauthorized f

Encapsulation Theorem

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Any unlinkable-HCCA + (plain) CCA = rerandomizable RCCA

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Proof.

Encapsulate CCA scheme inside *any* unlinkable HCCA scheme

- New scheme inherits outer unlinkability
- Inner CCA scheme "cancels" everything except identity function

□

Outline

Introduction
- Opposing Demands for Encryption

Security Defs
- Homomorphic CCA

Relationships among Definitions

Construction

Conclusion
- Summary
- Open problems

Construction

Parameterized family of constructions achieving our definitions:

- Message space: $\mathcal{G}^n$, where $\mathcal{G}$ is cyclic group.
- $\mathcal{H}$ is any subgroup of $\mathcal{G}^n$.
- Allowed transformations: $m \mapsto f * m$, for all $f \in \mathcal{H}$.

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Example instantiations:

- Allow all group operations in $\mathcal{G}^n$
- Allow only "scalar multiplication" of vectors:

$$(m_1, \ldots, m_n) \mapsto (f \cdot m_1, \ldots, f \cdot m_n)$$

- Allow group operations only on particular components – other components non-malleable
- Allow only identity function (Rerandomizable RCCA)

Construction

Our construction significantly generalizes rerandomizable RCCA scheme of [PR07].

- Obtain [PR07] scheme as special case
- Uses techniques from [G+04,CS01].

Theorem

Our construction is unlinkable & HCCA-secure under DDH assumption in 2 groups of related size.

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Our contributions:

- New definitions for case of unary homomorphic encryption
- Justify definitions by relating to existing ones
- Family of constructions that achieve definitions

Open problems

Extend to binary operations: $\mathsf{Enc}(\alpha), \mathsf{Enc}(\beta) \rightsquigarrow \mathsf{Enc}\big(f(\alpha, \beta)\big)$

- We show that natural generalization is impossible!
- Some slight relaxation possible (work in progress)
- Even new *security definitions* would be non-trivial.

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"Key-activated" homomorphic encryption:

- Scheme is CCA secure ...
- ... unless you have a token that "activates" only selected homomorphic features.

takk fyrir.*

*: *Thank you* (Icelandic)